BOUNDARY CONDITIONS AT THE SURFACE OF AN IRRADIATED SEMIBOUNDED MASS

D. A. Kurtener and A. F. Chudnovskii

Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 1, pp. 124–128, 1968

UDC 536.3

A method is proposed for formulating the boundary conditions at the surface of a semibounded mass naturally exposed to solar radiation and in contact with the surrounding atmosphere.

Under natural conditions, heat transfer at a surface takes place in several ways at the same time (by convection, radiation, evaporation). In such cases the formulation of the boundary conditions at the surface in one-dimensional problems of nonstationary heat transfer presents certain difficulties.

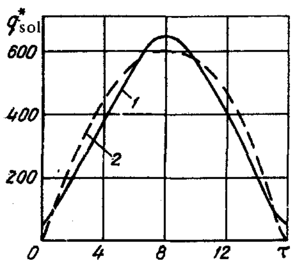

Fig. 1. Empirical curve (1) of diurnal variation in solar radiation q^*_{sol} (kcal/m^2 · hr) [7] and its approximation using (1): $q^*_{sol} = -9.4\tau^2 + 150\tau$ (curve 2). The initial time ($\tau = 0$) corresponds to 4 a.m.

A method of taking into account heat-transfer conditions at the surface based on an extension of the idea proposed in [1–3] is described below.

Under natural conditions the air temperature, the wind speed, the solar radiation, the albedo, the temperature of objects with which the surface is involved in radiative heat exchange (T_w), and other factors varying continuously with time determine the nature of surface heat transfer.

An analysis of the experimental data shows that two cases can be distinguished:

1. During the time interval τ_f none of the factors determining the heat-transfer conditions at the surface vary significantly. Consequently, in first approximation they are constant and their values averaged over τ_f can be used in the calculations.

2. During the interval τ_f certain of these factors vary along curves that have a single maximum or minimum. In first approximation they are represented by a quadratic trinomial

$$y = V_1\tau^2 + V_2\tau + V_3, \qquad (1)$$

where V_1, V_2, and V_3 are constants and $0 \le \tau \le \tau_f$. For example, the quantities q^*_{sol} (Fig. 1) and T_a (Fig. 2) are well described by (1).

Thus, in the second case we assume that some factors are constant and others vary in accordance with (1).

In the general case the heat balance equation for the surface is

$$q_s = q_{sol} - q_{res} - q_c - q_e. \qquad (2)$$

Heat-transfer theory provides various formulas from which the quantities on the right-hand side of Eq. (2) can be calculated with varying accuracy for specific conditions. Thus,

$$q_{sol} = (1-r)\,q^*_{sol}, \qquad (3)$$

$$q_c = \alpha\,[T(0,\tau) - T_a], \qquad (4)$$

where $T(0,\tau)$ is the surface temperature at time τ.

The evaporation losses q_e depend on the difference of the absolute humidity of the air at two heights ΔE and on the wind speed v. In accordance with [4],

$$q_e = \zeta v \Delta E, \qquad (5)$$

where ζ is a coefficient.

The radiant heat flux q_{res} is usually determined from the Stefan-Boltzmann law

$$q_{res} = \varepsilon\sigma\left\{\left[\frac{T(0,\tau)}{100}\right]^4 - \left(\frac{T_w}{100}\right)^4\right\}. \qquad (6)$$

Using the Chebyshev method of best approximation, we replace $\left[\frac{T(0,\tau)}{100}\right]^4$ and $\left(\frac{T_w}{100}\right)^4$ with binomials of the form $nT - p$. When $T(0,\tau)$ and T_w vary over the same temperature range, the coefficients n and p are respectively equal and expression (6) is written in the form

$$q_{res} = n\varepsilon\sigma\,[T(0,\tau) - T_w]. \qquad (7)$$

This transformation is very effective in connection with radiative heat transfer under natural conditions, where the range of possible values of $T(0,\tau)$ and T_w is not large. Thus, for the temperature interval (268–308)° K the error due to this transformation does not exceed 7%, and n = 0.96.

We substitute expressions (3), (4), (5), and (7) into (2). For the first case considered all the quantities in these equations are constant, then

$$q_s = M - NT(0,\tau), \qquad (8)$$

where

$$N = \alpha + n\varepsilon\sigma, \qquad (9)$$

$$M = (1-r)\,q^*_{sol} + \alpha T_a + n\varepsilon\sigma T_w + \zeta v \Delta E. \qquad (10)$$

The heat flow q_s through the surface is given by

$$q_s = -\lambda\,\frac{dT(0,\tau)}{dx}. \qquad (11)$$

Consequently, the boundary conditions are written as

$$-\lambda \frac{dT(0,\ \tau)}{dx} = N\left[\frac{M}{N} - T(0,\ \tau)\right]. \qquad (12)$$

Relation (12) is formally identical to the boundary conditions of the third kind. Consequently, we can use all those solutions of the heat conduction equation obtained for boundary conditions of the third kind. This is one of the principal advantages of the proposed method of formulating the boundary conditions at the surface.

In the second case as indicated above, some of the quantities determining the heat transfer conditions vary on the calculation interval. We represent these quantities by means of (1) and then substitute the results in (3), (4), (5), and (7). Solving, as before, the heat balance equation (2) and using (11), we obtain an expression for the boundary conditions at the surface in the form

$$-\lambda \frac{dT(0,\ \tau)}{dx} = A\tau^2 + B\tau + C - DT(0,\ \tau), \qquad (13)$$

where A, B, C, and D are constants.

No published solutions of the heat conduction equation with condition (13) are known to the authors. However, the relatively simple form of (13) permits us to find solutions of the heat conduction equation using the Laplace transform method. Thus, for a semibounded mass the solution of

$$\frac{\partial^2 T(x,\ \tau)}{\partial x^2} - \frac{1}{a}\frac{dT(x,\ \tau)}{\partial \tau} = 0 \qquad (14)$$

with condition (13) and a uniform initial distribution $T(x, 0) = T_0$ has the following form:

$$T(x,\ \tau) - T_0 = \frac{2A\lambda^2}{a^2D^5}\Bigg[-\exp(Hx + H^2 a\tau)\times \operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}} + H\sqrt{a\tau}\right) + \sum_{\xi=0}^{4}(-2H\sqrt{a\tau})^{\xi}\, i^{\xi}\operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}}\right)\Bigg] + \frac{B\lambda^2}{aD^3}\Bigg[-\exp(Hx + H^2 a\tau)\times \operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}} + H\sqrt{a\tau}\right) + \sum_{\xi=0}^{2}(-2H\sqrt{a\tau})^{\xi}\, i^{\xi}\operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}}\right)\Bigg] + \left(\frac{C}{D} - T_0\right)\Bigg[-\exp(Hx + H^2 a\tau)\times \operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}} + H\sqrt{a\tau}\right) + \operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}}\right)\Bigg],$$

where

$$H = \frac{D}{\lambda}, \quad \operatorname{erfc} U = \frac{2}{\sqrt{\pi}}\int_U^{\infty} \exp(-v^2)\, dv,$$

$$i^{\xi}\operatorname{erfc} U = \int_U^{\infty} i^{\xi-1}\operatorname{erfc} z\, dz.$$

The functions erfc U and i^{ξ}erfc U have been tabulated; their values are given in [5].

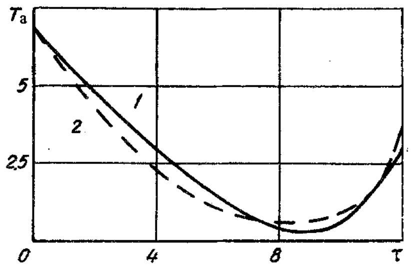

Fig. 2. Empirical curve (1) of air temperature (deg) according to the data of [6] and its approximation using (1): $T_a = 0.1\tau^2 - 1.5\tau + 7$ (curve 2). The initial time corresponds to 7 p.m.

Thus, the essence of the proposed method consists of examining the heat balance equation for the surface with the object representing the heat flow through that surface either as a linear function of the surface temperature (case 1) or as the sum of a linear function of the surface temperature and a quadratic function of time (case 2). This is achieved by selecting and transforming the formulas for the components of the heat balance equation (2) so that only the first power of the surface temperature is represented in those formulas, while the time-varying quantities can be represented by means of a quadratic trinomial.

NOTATION

T is the air temperature; v is the wind speed; q^*_{sol} is solar radiation on a horizontal surface; r is the albedo; λ is the thermal conductivity; a is the thermal diffusivity; σ is the Stefan-Boltzmann constant; ε is the reduced emissivity; α is the convective heat-transfer coefficient; q_s is the heat flow through the surface; q_{sol} is the solar radiation heat flux; q_{res} is the resultant radiative heat flux; q_c is the convective heat flux; and q_e are evaporative heat losses.

REFERENCES

1. B. P. Aleksandrov and A. V. Kurtener, Physical Basis of the Soil Heat Balance [in Russian], Leningrad, 1935.
2. A. F. Chudnovskii, Collection of Papers on Agrophysics [in Russian], Moscow, no. 4, 1948.
3. I. A. Ioffe and A. P. Shirobokova, Doklady VASKhNIL, no. 3, 1966.
4. R. Slater and N. McIlroy, Practical Microclimatology [Russian translation], izd. Progress, Moscow, 1964.
5. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids [Russian translation], Moscow, 1964.
6. K. I. Budaeva and G. G. Semikina, Collection of Papers on Agrophysics [in Russian], Leningrad, no. 12, 1965.
7. K. Ya. Kondrat'ev, The Radiant Energy of the Sun [in Russian], Leningrad, 1954.

21 February 1967 Leningrad Agrophysics Institute